WHY ANOVA?

- Student t-statistics are indispensable tools for comparing **two** population means. What if we need to compare three or more means?

WHY ANOVA?

- Student t-statistics are indispensable tools for comparing **two** population means. What if we need to compare three or more means?
  - If we want to see whether average income are different at four locations: North, South, East, and West, how many t-test do we need to run?

WHY ANOVA?

- Student t-statistics are indispensable tools for comparing **two** population means. What if we need to compare three or more means?
  - If we want to see whether average income are different at four locations: North, South, East, and West, how many t-test do we need to run?
  - $H_0 : \mu_N = \mu_S = \mu_E = \mu_W$

WHY ANOVA?

- Student t-statistics are indispensable tools for comparing **two** population means. What if we need to compare three or more means?
  - If we want to see whether average income are different at four locations: North, South, East, and West, how many t-test do we need to run?
  - $H_0 : \mu_N = \mu_S = \mu_E = \mu_W$
  - Assume that for each t-test, we correctly reject H_0 when it's false with probability 0.95. What is the probability that all of the pairwise t-tests correctly reject H_0 when it's false with probability 0.95?

WHY ANOVA?

- Student t-statistics are indispensable tools for comparing **two** population means. What if we need to compare three or more means?
  - If we want to see whether average income are different at four locations: North, South, East, and West, how many t-test do we need to run?
  - $H_0 : \mu_N = \mu_S = \mu_E = \mu_W$
  - Assume that for each t-test, we correctly reject H_0 when it's false with probability 0.95. What is the probability that all of the pairwise t-tests correctly reject H_0 when it's false with probability 0.95?
  - The probability of making type I error now is $1 - 0.95^6 \approx 0.265$

WHY ANOVA?

- Student t-statistics are indispensable tools for comparing **two** population means. What if we need to compare three or more means?
  - If we want to see whether average income are different at four locations: North, South, East, and West, how many t-test do we need to run?
  - $H_0 : \mu_N = \mu_S = \mu_E = \mu_W$
  - Assume that for each t-test, we correctly reject H_0 when it's false with probability 0.95. What is the probability that all of the pairwise t-tests correctly reject H_0 when it's false with probability 0.95?
  - The probability of making type I error now is $1 - 0.95^6 \approx 0.265$
- We cannot carry out multiple t-tests because it inflates the type I error.

Assumptions of ANOVA

- The data should be from a normally distributed population
- The variance in each experimental condition is fairly similar (homogeneity of variance)
- Observations should be independent and random
- The dependent variable should be measured on at least an interval scale.

- Hypotheses:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_c$$
$$H_1 : \text{Not all population means are equal}$$

- Test Statistics: $F = \frac{MSB}{MSW} = \frac{SSB/(c-1)}{SSW/(n-c)}$ where
  - MSB: Mean square between groups
  - MSW: Mean square within groups
  - SSB: Sum square between groups
  - SSW: Sum square within groups
  - SST: Total sum square
  - $SST = SSB + SSW$
- Decision Rule: Reject H_0 if $F > F_u = F_{\alpha,c-1,n-c}$

- Critical range

$$Q_u \times \sqrt{\frac{MSW}{2}(\frac{1}{n_j} + \frac{1}{n_{j'}})}$$

$$Q_u = Q_{\alpha,c,n-c}$$

- Calculate $|\bar{X}_j - \bar{X}_j'|$
- If $|\bar{X}_j - \bar{X}_j'| >$ Critical range, then $\mu_j \neq \mu_{j'}$
- Number of pairs is $\dfrac{c(c-1)}{2}$

Randomised Block Design

- This is an experimental technique where data in groups are divided into fairly homogeneous subgroups called blocks to remove variability from random error.
- If we discover that there is a significant treatment effect in a randomised block design, then the next step is to make comparisons between means using the Tukey procedure.

- n observations, c groups, r blocks
- Hypotheses:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_r$$
$$H_1 : \text{Not all population means of r blocks are equal}$$

- Test Statistics: $F = \frac{MSBL}{MSE} = \frac{SSBL/(r-1)}{SSE/(c-1)(r-1)}$ where
  - $MSBL$: Mean square between block
  - SSE: Sum square errors
  - MSE: Mean square errors
  - $SSBL$: Sum square between blocks
  - $SSE = SST - SSBL - SSB$
- Decision Rule: Reject H_0 if $F > F_u = F_{\alpha,r-1,(c-1)(r-1)}$

- Hypotheses:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_r$$
$$H_1 : \text{Not all population means of c groups are equal}$$

- Test Statistics: $F = \dfrac{MSB}{MSE} = \dfrac{SSB/(c-1)}{SSE/(c-1)(r-1)}$
- Decision Rule: Reject H_0 if $F > F_u = F_{\alpha,c-1,(c-1)(r-1)}$

- Critical range

$$Q_u \times \sqrt{\frac{MSE}{2}}$$
$$Q_u = Q_{\alpha,c,(c-1)(r-1)}$$

- Calculate $|\bar{X}_j - \bar{X}_j'|$
- If $|\bar{X}_j - \bar{X}_j'| >$ Critical range, then $\mu_j \neq \mu_{j'}$